Parasitic Momentum Flux in the Tokamak Core

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Careful geometric analysis shows that energy transfer from the electrostatic potential to ion parallel flows breaks symmetry in the fully nonlinear toroidal momentum transport equation, causing countercurrent rotation peaking without applied torque.

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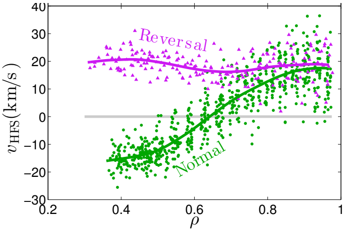
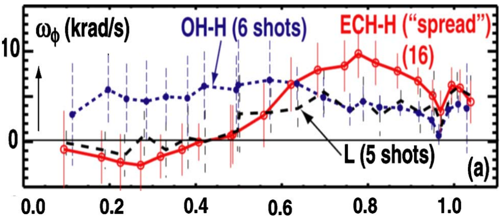
Outline

- Background
  - Experiment:
    - Intrinsic rotation and rotation reversals
  - Theory:
    - Intrinsic rotation: Vanishing momentum flux
- Rotation model
  - Intuitive cartoon of axisymmetric example
  - Symmetry constraints on nonaxisymmetric equations
  - Free-energy flow in phase space$\Rightarrow$momentum flux
  - Predicted core rotation peaking:
    scaling, behavior, and experimental comparison

Tokamak plasmas rotate spontaneously without applied torque.





TCV Ohmic shots (I_p ≈155, 195kA)

Stoltzfus-Dueck et al PoP '15

DIII-D Ohmic and ECH shots

deGrassie et al PoP '07

Important for stability against resistive wall modes at low torque (ITER).

Typical intrinsic rotation profiles have three regions:

- Edge: Co-rotating (due to ion orbit shifts)
- Mid-radius "gradient region": Countercurrent peaking or ∼flat
  - Gradient exhibits sudden 'reversals' at critical parameter values.
  - Rotation profiles often pass through zero.
- Sawtoothing region inside $q = 1$: Flat or weak cocurrent peaking

In axisymmetric geometry, neoclassical momentum transport is negligible.

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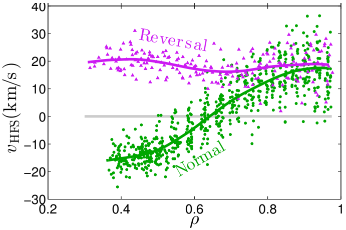

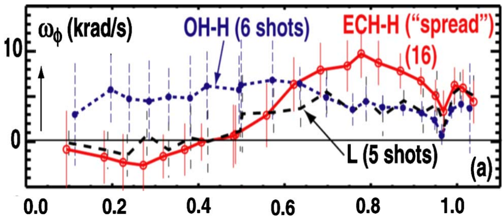

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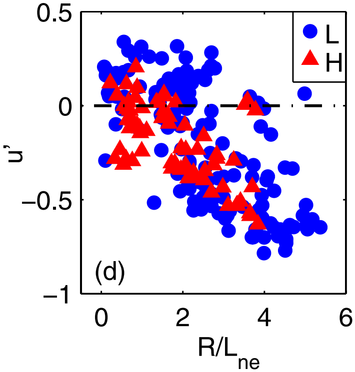
AUG measurements indicate correlation of density peaking and counter-current momentum peaking.

AUG intrinsic rotation database
(Angioni PRL '11⇒)

- >200 intrinsic rotation profiles
- Ohmic, L-, and H-modes
- With & without ECH & ICH
- $u' \doteq -(R/v_{ti})du_\varphi/dr$



Across all discharge types, density peaking correlates with countercurrent rotation peaking.

Today's model offers a simple explanation for this. Stay tuned...

Intrinsic rotation profiles result from vanishing momentum flux.

Axisymmetric steady state with no torque⇒zero momentum outflux:

$$0 = \Pi = -\nu \nabla L + v_{\text{pinch}} L + \Pi^{\text{res}} \Longrightarrow \nabla L = (v_{\text{pinch}} L + \Pi^{\text{res}})/\nu$$

Toroidal momentum gradient ∇L is set by balancing

- Viscous flux ($-\nu \nabla L$) (saturation) against both
- Momentum pinch ($v_{\text{pinch}} L$) due to
  - 'Turbulent equipartition' due to ∇B (Hahm et al PoP '07)
  - Coriolis force (Peeters et al PoP '09)
- Residual stress (Π^{res}, independent of L)
  - Only explanation for peaked profiles that cross $L = 0$

More than one mechanism may be important for a given discharge.

Solve for $\nabla L = (v_{\text{pinch}} L + \Pi^{\text{res}})/\nu$, summing over all spin-up terms.

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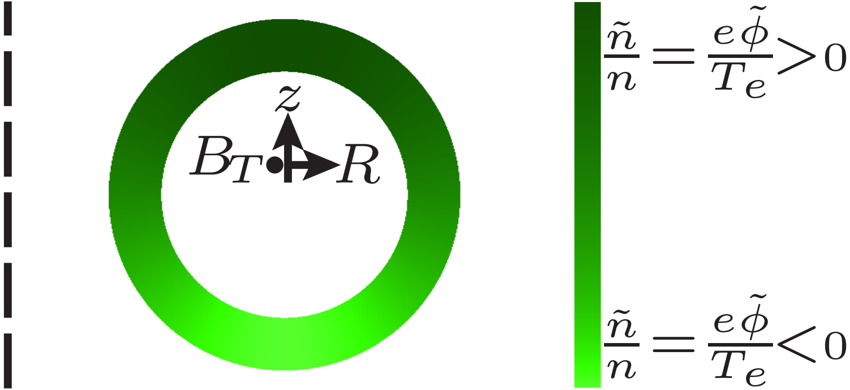
What drives symmetry-breaking momentum flux Π^{res},
in the absence of rotation and of rotation shear?

Dual role for slowly varying $\partial_\theta \phi$ causes countercurrent peaking.



I. Example: axisymmetric ($n = 0$), low-frequency density fluctuations.
$E_\| = -b_p(\partial_\theta \phi)/r$ accelerates ions out of density hump.
$E_\| u_{\| i} = -b_p u_{\| i}(\partial_\theta \phi)/r$ transfers energy to ion parallel flows.

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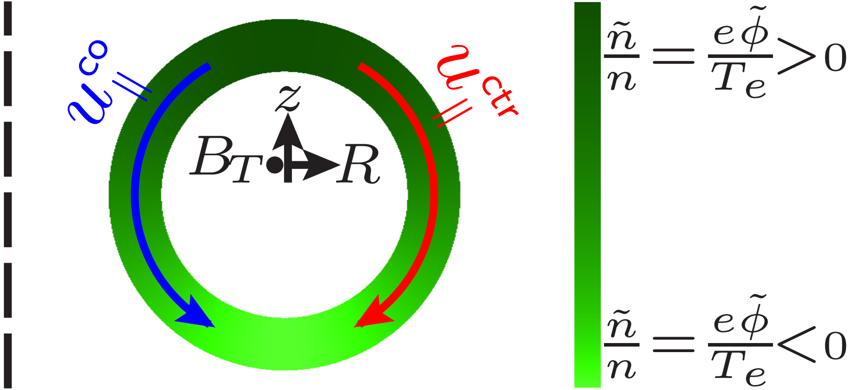

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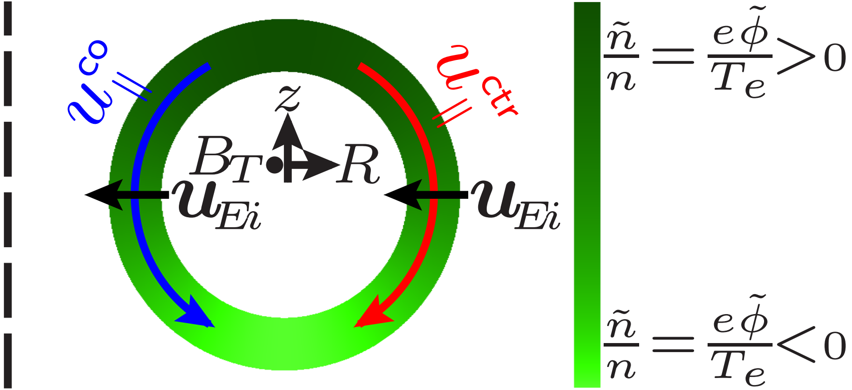

II. Weak radial $\boldsymbol{E} \times \boldsymbol{B}$ drift $u_{Ei}^r = -(cb_T/Br)\partial_\theta \phi$ advects ions.
Outflow of cocurrent momentum: $\Pi = [-cb_T(\partial_\theta \phi)/Bb_p] m_i n_{i0} R b_T u_{\parallel i}$
Momentum flux$\propto$energy transfer because $E_\parallel / b_p = -\partial_\theta \phi = E_\perp / b_T$.

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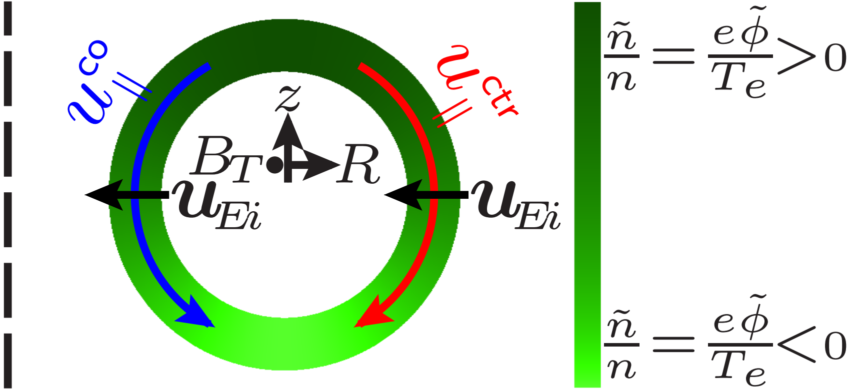


III. Slow poloidal potential variation in $\partial_\theta \phi \sim k_\parallel \phi / b_p \sim \phi / r$:

- neglected by fluxtube orderings, but
- breaks symmetry because $\hat{b}$ neither parallel nor perp to $\hat{\varphi}$.

Symmetry restricts contributions to residual stress.

In the simplest radially local fluxtube limit with

- up-down symmetric magnetic geometry,
- no background rotation or rotation shear, and
- no background $\boldsymbol{E} \times \boldsymbol{B}$ shear,

the delta-f gyrokinetic equations satisfy a symmetry:

If $f(\rho, \vartheta, \xi, v_\parallel, \mu, t)$, $\phi(\rho, \vartheta, \xi, t)$ is a solution
so is $-f(-\rho, -\vartheta, \xi, -v_\parallel, \mu, t)$, $-\phi(-\rho, -\vartheta, \xi, t)$,

with opposite sign of the dominant toroidal momentum flux.
(Peeters and Angioni PoP '05, Peeters et al NF '11)

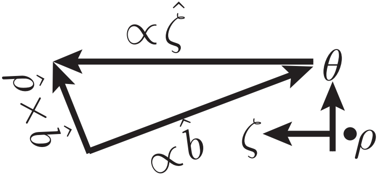
This implies: toroidal momentum flux should vanish for terms that flip sign (but does *not* imply that invariant terms *must* drive momentum flux).

The radial $\boldsymbol{E} \times \boldsymbol{B}$ drift with true $\nabla_\perp \phi$ breaks the symmetry.

Define convenient directions

$$\hat{\rho} \doteq \frac{\nabla \rho}{|\nabla \rho|}, \quad \hat{\rho} \doteq \hat{\zeta} \times \hat{\rho}$$

and decompose $\hat{b} = b_T \hat{\zeta} + b_p \hat{\rho}$.



Use $\hat{\rho} \times \hat{b} = (\hat{\zeta} - b_T \hat{b})/b_p$ to evaluate

$$\boldsymbol{u}_{Ei} \cdot \hat{\rho} = \frac{c}{B} \hat{b} \times \nabla \phi \cdot \hat{\rho} = \frac{c}{B} \hat{\rho} \times \hat{b} \cdot \nabla \phi = \frac{c}{b_p B} \hat{\zeta} \cdot \nabla \phi \overbrace{- \frac{c b_T}{b_p B} \hat{b} \cdot \nabla \phi}^{u_{Ei}^{(2)}} .$$

Symmetry prevents first term $\propto \hat{\zeta} \cdot \nabla \phi \propto \partial_\xi \phi$ from driving residual stress.

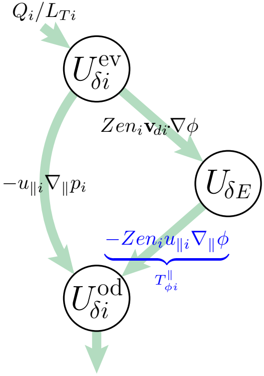
Second term cancels the parallel gradient included in $\hat{\zeta} \cdot \nabla \phi \neq \rho \times \hat{b} \cdot \nabla \phi$:

- Nominally smaller than the first term, by $k_\parallel / k_\perp b_p$, but
- Contributes a symmetry-breaking term to momentum flux $m_i n_{i0} b_T R_0 u_{Ei}^x u_{\parallel i}$:

$$\Pi_\varphi^{(2)} = n_{i0} m_i R_0 b_T u_{\parallel i} u_{Ei}^{(2)} = -(c m_i n_{i0} R_0 / b_p B_0) u_{\parallel i} \nabla_\parallel \phi$$

If ion parallel flows are excited, co-current momentum flows out.

Turbulence fluctuation amplitude is regulated by free-energy balance:



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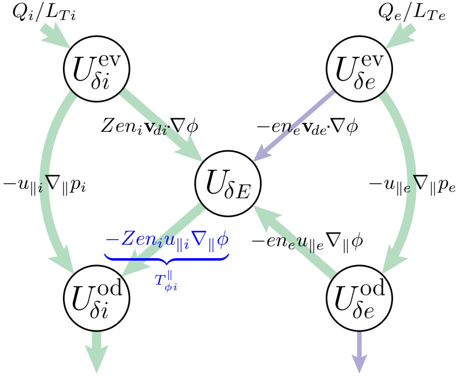


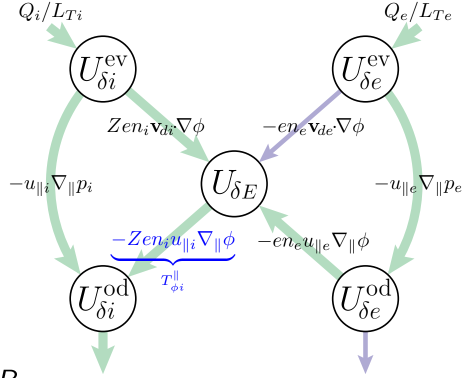
If ion parallel flows are excited, co-current momentum flows out.

Turbulence fluctuation amplitude is regulated by free-energy balance:

Counter-current peaking due to ion Landau damping $T^{\parallel}_{\phi i} > 0$, if

$$\omega \lesssim v_{ti}/qR,$$

as seen in Wang/Grierson simulations.



$$\Pi^{(2)}_{\varphi} = -\frac{cm_i n_{i0} R_0}{b_p B_0} u_{\parallel i}\nabla_{\parallel}\phi = \frac{R_0}{\Omega_{ci\theta}} T^{\parallel}_{\phi i}$$

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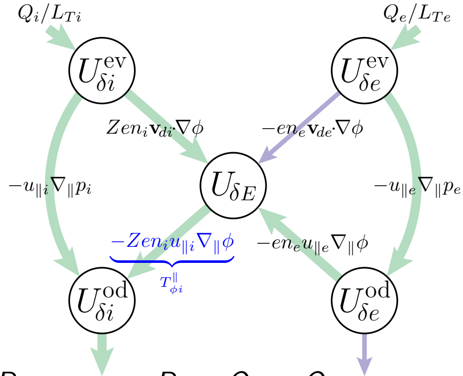


Let a fraction $0 \leq f_L \leq 1$ of turbulent free energy pass through $T^{\parallel}_{\phi i}$, then residual stress may be solved for as:

$$\Pi^{(2)}_{\varphi} = -\frac{cm_in_{i0}R_0}{b_pB_0}u_{\parallel i}\nabla_{\parallel}\phi = \frac{R_0}{\Omega_{ci\theta}}T^{\parallel}_{\phi i} = f_L\frac{R_0}{\Omega_{ci\theta}}\left(\frac{Q_i}{L_{Ti}} + \frac{Q_e}{L_{Te}}\right)$$

When ion Landau damping is significant, one obtains counter-current rotation peaking with a simple scaling.

Now balance viscous momentum flux and residual stress:

$$-\chi_\varphi \frac{n_{i0} m_i R_0 u_\varphi}{L_\varphi} = \Pi_\varphi^{(2)} = f_L \frac{R_0}{\Omega_{ci\theta}} \Big(\chi_i \frac{n_{i0} T_{i0}}{L_{Ti}^2} + \chi_e \frac{n_{e0} T_{e0}}{L_{Te}^2}\Big), \text{ so that}$$

$$\frac{u_\varphi}{v_{ti}} = -f_L \rho_{i\theta} \Big(\frac{\chi_i}{\chi_\varphi} \frac{L_\varphi}{L_{Ti}^2} + \frac{\chi_e}{\chi_\varphi} \frac{Z T_{e0} L_\varphi}{T_{i0} L_{Te}^2}\Big) \sim -f_L \Big(1 + \frac{Z T_{e0}}{T_{i0}}\Big) \frac{\rho_{i\theta}}{L_\perp},$$

where the last form took $\chi_\varphi \sim \chi_i \sim \chi_e$ and $L_\varphi \sim L_{Ti} \sim L_{Te} \doteq L_\perp$.

With Ampere's Law $2\pi r B_p \sim 4\pi I_p / c$, we get the dimensional estimate

$$\boxed{u_\varphi \sim -5 f_L \frac{(T_{i0} + Z T_{e0})(\text{keV})}{Z I_p(\text{MA})} \frac{r}{L_\perp} \text{km/s},}$$

comparable with peaking measured on DIII-D, JET, C-mod, AUG, and TCV.

Density peaking and counter-current momentum peaking are both active only for low-frequency turbulence.

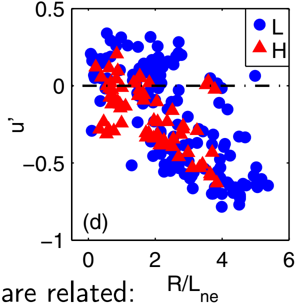
Particle pinch due to electron precession, active when frequency satisfies

$$\omega \lesssim \omega_{de} \sim k_\perp v_{te} \rho_e / R$$

(Angioni et al NF'12)

Counter-current peaking due to ion Landau damping, active when

$$\omega \lesssim v_{ti} / qR$$



Although physical mechanisms are distinct, criteria are related:

$$\frac{\omega_{de}}{v_{ti}/qR} = \frac{Z T_{e0}}{T_{i0}} q k_\perp \rho_i$$

Both will be active during low-frequency turbulence $\omega \lesssim \omega_{de} \sim v_{ti}/qR$.

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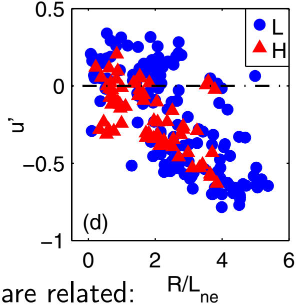

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DIII-D can directly test the theory by comparing mid-gradient rotation peaking with fluctuation frequencies (vs v_{ti}/qR).

Summary

A geometrically higher-order portion of the $\boldsymbol{E} \times \boldsymbol{B}$ drift causes a nondiffusive momentum flux that:

- results from symmetry-breaking by excitation of ion parallel flows
  - does not require $\langle u_\varphi \rangle$ or $\nabla \langle u_\varphi \rangle \Rightarrow$residual stress
  - a fully nonlinear mechanism, not quasilinear
- causes counter-current rotation peaking in the core
- drives experimentally relevant rotation peaking around

$$u_\varphi \approx -5 f_L \frac{(T_{i0} + ZT_{e0})(\mathrm{keV})}{ZI_p(\mathrm{MA})} \frac{r}{L_\perp} \mathrm{km/s},$$

- acts only when turbulence is at low enough frequencies to excite ion parallel flows
  - consistent with Wang/Grierson simulations
  - $\sim$same criterion as for density peaking, consistent with observed relation of density and rotation peaking across many discharge types
  - opportunity for a direct experimental test